Zonal Informatics Olympiad, 2002

Time: 3 hours *February 10, 2002*

Attempt all questions.

1. Consider a plate stacked with several disks, each of a different diameter (they could all be, for instance, *dosas* or *chapatis* of different sizes). We want to arrange these disks in decreasing order according to their diameter so that the widest disk is at the bottom of the pile and every disk is smaller than the disks below it.

   The only operation available for manipulating the disks is to pick up a stack of them from the top of the pile and invert that stack. (This corresponds to lifting up a stack *dosas* or *chapatis* between two big spoons and flipping the stack.)

   (a) Describe a precise method (algorithm) to sort the disks using this *flip* operation.

   (b) What initial arrangement of n disks will force us to use the "flip" operation the maximum number of times before the entire pile is stacked up correctly?

2. A new test for tuberculosis is found to be 80% accurate—if a person has the disease, the test is positive 80% of the time and if a person does not have the disease, the test is negative 80% of the time. A person is picked at random from a group in which 10% of the people actually have tuberculosis.

   (a) If the randomly selected person's test result is positive, what is the probability that he has tuberculosis?

   (b) If the randomly selected person's test result is negative, what is the probability that he has tuberculosis?

3. In a normal pan balance (as used by, say, a vegetable vendor) the item to be bought is balanced against a known combination of weights. The weights may be applied on either side of the pan—for instance, to weigh 1500 g of onions, the vendor may either place the onions in one pan and a 1000 g and 500 g weight in the other, or place the onions along with a 500 g weight in one pan and a 2000 g weight in the other pan.

   We would like to be able to weigh all quantities between 1 g and 4000 g to the nearest gram using such a balance. One way to do this is to have 12 weights weighing 1 g, 2 g, 4 g, 8 g, 16 g,. . . , 2048 g.

   (a) Design a smaller set of weights to achieve the same objective.

   (b) In general, to weigh items between 1 g and n g to the nearest gram, what is the smallest sequence of weights we can use (in terms of n)?

4. (a) Let A be a matrix of integers. Suppose we first arrange each row in ascending order and then arrange each column in ascending order. Show that the rows remain in ascending order after rearranging the columns.

   (b) Let A be an $n \times n$ matrix of integers such that each row *and* each column is arranged in ascending order. we are told that a number k appears in A and we would like to find out its position—that is, the row i and column j such that $A(i, j) = k$.

      (i) Show that we can do this by examining at most $2n$ values in A.

      (ii) In what position should k be to force us to look at $2n$ values in A?

5. We have n balls that look identical but each of them has a different weight. We have a balance in which we can weigh one ball against another ball.

   (a) Describe an efficient method to find the heaviest ball. How many weighings does the method require, in terms of n?

   (b) One way to find the second heaviest ball is to first find the heaviest ball using the method from part (a), remove it from the collection, and then find the heaviest among the remaining balls using the same method. Describe a more efficient method to find the second heaviest ball. How many weighings does the improved method require, in terms of n?

6. We have a bucket full of marbles, some of which are black and some of which are white. We also have a big sack with an unlimited supply of black marbles.

   We keep repeating the following step:

   - We close our eyes and pick out two marbles from the bucket at random.
   - If both marbles are of the same colour, we throw away both of them and put a fresh black marble from the sack into the bucket.
   - If the marbles are not of the same colour, we throw away the black marble and replace the white marble back in the bucket.

   With each step, the number of marbles in the bucket decreases by one. Eventually, we are left with a single marble in the bucket. What is the relationship, if any, between the colour of the last marble and the contents of the bucket that we started with?

Zonal Informatics Olympiad, 2002, Part A

Model Solutions

1. (a) Find the largest disk and flip the top of the stack upto and including this disk. This will take the largest disk to the top of the pile. Then flip the entire stack to get the largest disk to the bottom.

   At step k+1, inductively the k largest disks are arranged in order at the bottom of the stack. Pick the largest disk above these k disks, flip the stack above this disk to take it to the top and then flip the top n-k disks to bring it to position k+1 from the bottom.

   (b) Let the disks be numbered 1,2,...,n, where disk 1 is smaller than disk 2 is smaller than ... disk n. In the worst case it takes two flips for each of the disks n, n-1, n-2, ...,3. After disks 3 to n are in the correct position, in the worst case 1 and 2 are inverted so we need one more flip. So, in the worst case, we need 2(n-2)+1 = 2n-3 flips.

```
For 2 disks, the worst case is   2
                                 1
Call this arrangement s2
```

   For 3 disks, the worst case is to get to s2 after 3 comes to the bottom. We would like to take 2 flips to get 3 to the bottom, so, working backwards,

```
2       3       1
1  <-   1  <-   3  = s3
3       2       2
```

   For 4 disks, we want s1 above 4 after flipping twice to get 4 to the bottom, so s4 is the last sequence of the following

```
1       4       3
3  <-   2  <-   2  = s4
2       3       4
```

```
4       1       1
```

For k+1 disks, we want sk above disk k+1 after flipping twice to get k to the bottom, so s(k+1) is obtained as follows: (Notation: reading a sequence from top to bottom, head is the top element, tail is the rest, last is the bottom element, init is all but the bottom, flip is a function that inverts a list):

```
sk        (k+1)      flip (init(flip sk)) = tail sk
:    <-    :    <-   (k+1)            (k+1)
(k+1)     flip sk     last (flip sk)      head sk
```

2. Formally:

Conditional probability (Bayes):

Pr(A given B) = Pr(A and B) / Pr (B)

(a) B is "test is positive", A is "has TB"

Pr (A and B) is (80/100)*(10/100) = 8/100
Pr (B) is Pr(yes TB, test positive) + Pr(no TB, test positive)
= (10/100)*(80/100) + (90/100)*(20/100)
= 26/100

So Pr(A given B) is 8/26 = 4/13

(a) B is "test is negative", A is "has TB"

Using similar computation as in (a)

Pr (A and B) is 2/100
Pr (B) is 74/100
Pr(A given B) 2/74 = 1/37

Informally:

In 100 people, 10 have TB, 90 do not.
For the 10 who do, 8 test positive, 2 do not.
For the 90 who don't, 18 test positive, 72 do not.

(a) 8 of 26 people who test positive have TB
(b) 2 of 74 people who test negative have TB

3. Think of one pan as the 'positive' pan and the other as the 'negative' pan. If we put w1 in the positive pan and w2 in the negative pan, the net weight is w1 - w2. Thus, if we use weights w1, w2, ..., wn, we can write out the net weight as a "ternary" n-digit number using digits +1/0/-1.

From this, it follows that the optimal set of weights corresponds to powers of 3.

(a) 1, 3, 9, 27, 81, 243, 729, 2187, 720 (the last weight is 4000-3280, where 3280 is the sum of the previous 8 weights).

(b) In general, we would use 1, 3, 9, ..., log_3(n), n-(1+3+..+log_3(n))

4. (a) Look at A after rearranging the columns.

Consider any pair of adjacent columns k-1 and k. We show that for each row j, A(j,k-1) is smaller than A(j,k).

The first entry A(1,k) is bigger than A(1,k-1). This is because A(1,k) dominates at least one entry in column k-1 (the one that was originally to the left of it) and so dominates A(1,k-1), the minimum element in column k-1.

Now consider A(2,k). We can argue that A(2,k) dominates at least two entries in column k-1 and hence dominates A(2,k-1), the second smallest entry in column k-1.

Suppose A(2,k) dominates only one entry in column k-1 (it dominates at least one, the one originally to its left). Then, the element originally to the left of A(2,k) is the element currently at A(1,k-1). This means that A(1,k) originally had another element A(j,k-1) to its left, not A(1,k-1). But then, A(2,k) > A(1,k) > A(j,k-1) > A(1,k-1), which contradicts the assumption that A(2,k) dominates only one entry in column k-1.

In a similar manner, we can show that for each j, A(j,k)

dominates at least j entries in column k-1 and so
dominates A(j,k-1).

Since for each row j and each column k, A(j,k) is bigger than A(j,k-1), it follows that all the rows are still in ascending order.

(b)(i) Check k against the top right corner element A(1,n)

Case 1: If k is smaller than A(1,n) then k cannot appear in the last column (since A(1,n) is the smallest entry in that column) so we have to search for A in rows 1 to n, columns 1 to n-1.

Case 2: If k is bigger than A(1,n) then k cannot appear in the first row (since A(1,n) is the largest entry in that row) so we have to search for A in rows 2 to n, columns 1 to n.

In the next step, we check k against the top right corner of the remaining matrix of interest (the top right corner is A(1,n-1) if Case 1 holds and A(2,n) if Case 2 holds) and repeat the earlier analysis.

With each step we reduce the matrix of interest by pruning either one row or one column. Since we started with n rows and n columns, within 2n steps (actually 2(n-1) steps) we would have pruned the matrix down to a single row and column, i.e. a single element.

(ii) Since the search proceeds from the top right to the bottom left, the search goes on longest if the element is at the bottom left corner, A(n,1).

5. (a) There are two natural methods:

The first is to pick up a pair of balls, weigh them, retain the heavier. Then, pick up each of the other balls in turn, weigh them against the current "champion" and retain the heavier one as the new "champion". The ball that remains at the end is the heaviest.

The second method is to organize a knockout tournament.

Pair up all the balls and retain the heavier one from each pair. Then, pair up the winners of the first round and do the same. Repeat until a single winner is found.

Both methods require n-1 weighings.

(b) The tournament solution to (a) helps us find the second heaviest ball more efficiently than by naively repeating the procedure for (a).

In the overall tournament, the second heaviest ball would have beaten every ball other than the heaviest one. So, we only have to find the heaviest among the log n balls that "lost" to the heaviest ball to find the second heaviest.

6. In each round, the number of black balls can either decrease by 1 (if we pick up black-black or black-white) or increase by 1 (if we pick up white-white) so not much information can be gleaned from analysing the black ball count.

However, notice that white balls are eliminated in pairs. Thus, the parity (even/odd) of the white balls in the bag is an invariant. Eventually, we have a single ball (odd). Thus, the last ball is white if only if the bag originally had an odd number of white balls.

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Zonal Informatics Olympiad, 2002–2003

Instructions to candidates

1. The duration of the examination is $2\frac{1}{2}$ hours.
2. The question paper carries 75 marks, broken up into five questions of 15 marks each.
3. Attempt all questions. There are no optional questions.
4. Question 3 has negative marking. No other question has negative marking.
5. There is a separate *Answer Sheet*. To get full credit, you *must* write the final answer in the space provided on the Answer Sheet.
6. Write *only* your final answers on the Answer Sheet. Do *not* use the Answer Sheet for rough work. Submit all rough work on separate sheets.

Zonal Informatics Olympiad, 2002–2003

Questions

1. We are given n integers $x_1, x_2, ..., x_n$, where n is even. Suppose we group these n numbers into $n/2$ pairs and add up each of the pairs. The *weight* of this grouping is the maximum of these sums.

   For example, if the input numbers are 5, 7, 8, -2, 6, 4, 5, 2 and if they are paired up as (5,-2), (7,4), (5,6), (2,8) then the sums of the 4 pairs are 3, 11, 11, and 10. The weight is the maximum of $\{3, 11, 11, 10\}$ and is thus 11.

   For each of the following sets of integers, find a way of grouping them into pairs so that the weight is minimized. In your answer, list out the grouping and then indicate the weight.

   (a) 103, 24, 77, 65, 12, 108, 69, 25, 66, 83

   (b) 83, 112, -16, 72, 161, 75, 152, -23, 77, 247

   (c) 19, 81, 2, 41, 61, 59, 28, 69, 76, 88 *(15 marks)*

2. We start with a two digit positive integer and construct a sequence of two digit numbers as follows. Let the current number be x. If $2x$ is less than 100, then the next number in the sequence is $2x$. Otherwise the next number in the sequence is $2x - 100$.

   A number is said to be *good* if we can start with the number and get back to the same number later in the sequence. A number that is not good is said to be *bad*.

   For example, 20 is a good number, because the sequence starting with 20 is 20, 40, 80, 60, 20. So, after four steps, we get back to 20. However 10 is bad because starting from 10 we get the sequence 10, 20, 40, 80, 60, 20, ... in which 10 never reappears.

   What is the common property that is shared by the set of good numbers?

   (15 marks)

3. We are passing a sequence of plates from Atul to Zenobia. Each plate has a number painted on it and no two plates have the same number. We have a table in front of us on which we can temporarily store a single stack of plates. At each step, we are allowed to do one of the following:

   - Take a plate from Atul and pass it on immediately to Zenobia.
   - Take a plate from Atul and put it on top of the stack on the table.
   - If there is at least one plate in the stack on the table, take the topmost plate off the stack and pass it on to Zenobia.

   In this process, we can rearrange the plates that Atul gives us before passing them on to Zenobia. For instance, if the sequence of numbers on the plates given to us by Atul is 1,2,3,4, we can pass them onto Zenobia in the sequence 2,4,3,1 as follows:

- Take plate 1 from Atul and start a stack on the table.
- Pass plate 2 directly from Atul to Zenobia.
- Take plate 3 from Atul and stack it on top of plate 1.
- Pass plate 4 directly from Atul to Zenobia.
- Take plate 3 off the stack and pass it to Zenobia.
- Take plate 1 off the stack and pass it to Zenobia.

We say that an input sequence is *compatible* with an output sequence if it is possible to rearrange this input sequence to produce the output sequence. For instance, we just showed that the input sequence 1,2,3,4 is compatible with the output sequence 2,4,3,1.

Consider the following input and output sequences of plates.

Input sequences	*Output sequences*
(1) 5,8,10,3,2,9,7,6,4,1	(A) 6,2,7,4,5,8,3,10,9,4
(2) 10,1,9,6,5,4,2,8,3,7	(B) 3,9,7,2,10,6,1,4,8,5
(3) 7,6,2,5,3,4,8,10,4,9	(C) 9,5,6,1,2,4,10,7,3,8

Indicate the compatible pairs from the sets {1,2,3} and {A,B,C}. Note that it is possible for one input sequence to be compatible with more than one output sequence and vice versa. Also, there could be input sequences in {1,2,3} that are not compatible with *any* output sequence from {A,B,C} and vice versa. *(Note: If you mark an input-output pair as compatible when it is not, you get negative marks!)*

(15 marks)

4. A thief breaks into a shop selling exotic powdered spices (masalas). He has a sack with him in which he can carry away spices weighing upto W kg. For each spice, the thief knows how much of that spice powder is available in the shop and the total value of that spice powder. The problem is for the thief to decide how much of each spice to steal so that he maximizes the value of the spices that he carries away.

   For instance, suppose that the thief can carry away 20 kg (that is, $W = 20$) and there are three spices available—turmeric, cloves and mustard. There are 18 kg of turmeric with a total value of Rs 2400, 10 kg of cloves with a total value of Rs 1500 and 15 kg of mustard with a total value of Rs 1800. We can represent these values by the following table:

   $W = 20$

	turmeric	cloves	mustard
amount	18	10	15
value	2400	1500	1800

   If the thief fills his sack with all 18 kg of turmeric and 2 kg of mustard, he will walk away with spices worth Rs $2400 + (\frac{2}{15} \times 1800)$ = Rs $2400 + 240$ = Rs 2640.

Here are three situations in which the thief finds himself. Calculate the maximum value he can steal in each case. In your answer, give the breakup of what the thief carries away in his sack and the total value.

(a) $W = 20$

	A	B	C
amount	15	10	18
value	1800	1500	2400

(b) $W = 30$

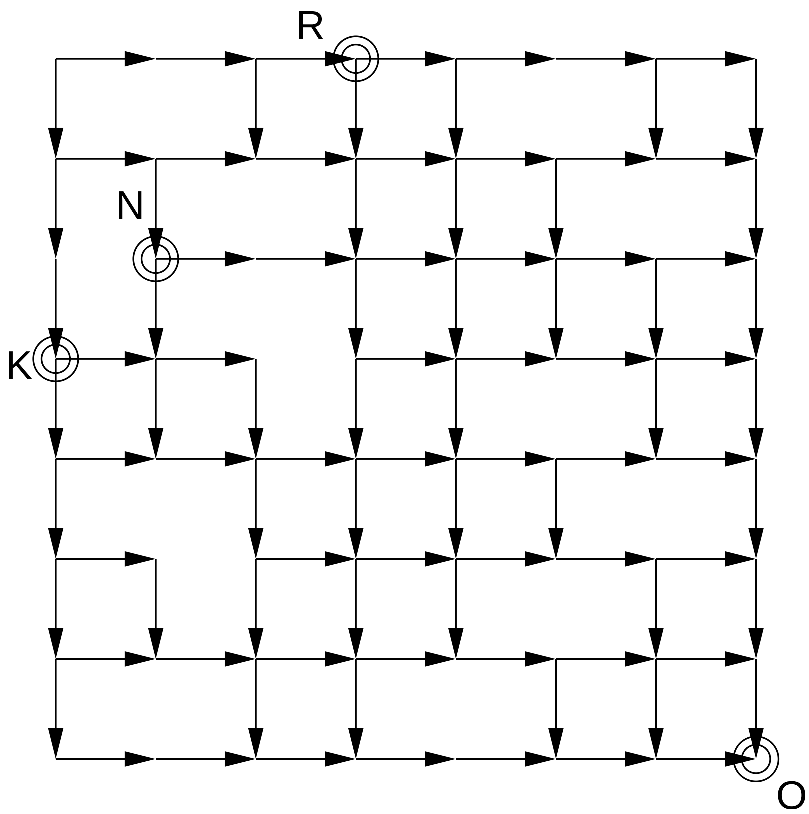
	A	B	C	D
amount	25	10	15	9
value	3000	1400	1800	1200

(c) $W = 30$

	A	B	C	D
amount	25	10	15	20
value	2250	1100	1500	1900

(15 marks)

5. Komal, Narain and Robert all work in the same office. All the roads in the city where they live are one-way and their route from home to office must take this into account. Here is a map of the city. The letters K, N and R mark the homes of Komal, Narain and Robert, respectively, and O marks the office where they work. The direction in which each road can be used is indicated by an arrow. For each person, calculate how many different routes that person can take from home to office.



(15 marks)

Zonal Informatics Olympiad, 2002–2003: *Answer sheet*

Roll No:	**Examination Centre:**

Important: *Write your final answers (and nothing else) in the space provided.*
Write all rough work on separate sheets.

1. (a) Grouping:
   Weight:

   (b) Grouping:
   Weight:

   (c) Grouping:
   Weight:

2.

3. Draw a line connecting each pair of compatible sequences: *(This question has negative marking!)*

Inputs	*Outputs*
(1)	(A)
(2)	(B)
(3)	(C)

4. (a) Sack contents:
   Total value:

   (b) Sack contents:
   Total value:

   (c) Sack contents:
   Total value:

5. (a) Komal: (b) Narain: (c) Robert:

For official use only. Do not write below this line.

1. | (a) | (b) | (c) |

2. | |

3. | + | − | Net |

4. | (a) | (b) | (c) |

5. | (a) | (b) | (c) |

| **Total** |

Zonal Informatics Olympiad, 2002–2003

Solutions

Explanations for the solutions are given separately, starting on the next page.

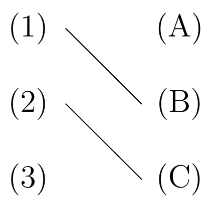
1. (a) Grouping: (12,108), (24, 103), (25,83), (65,77), (66, 69)
   Weight: 142

   (b) Grouping: (-23,247), (-16,161) (72,152) (75,112), (77,83)
   Weight: 224

   (c) Grouping: (2,88), (19,81), (28,76), (41,69), (59,61)
   Weight: 120

2. A number is good precisely when it is a multiple of 4.

3.



1 is compatible with B
2 is compatible with C
3 and A are not compatible with any sequence.

4. (a) Sack contents: 10 kg of B, 10 kg of C
   Total Value: 2833.33

   (b) Sack contents: 10 kg of B, 9 kg of D, 11 kg divided in any way between A and C.
   Total Value: 3920

   (c) Sack contents: 10 kg of B, 15 kg of C, 5 kg of D
   Total Value: 3075

5. (a) Komal: 116

   (b) Narain: 119

   (c) Robert: 97

Zonal Informatics Olympiad, 2002–2003

Solutions, with explanations

1. (a) Grouping: $\{(12, 108), (24, 103), (25, 83), (65, 77), (66, 69)\}$
   Weight: 142

   (b) Grouping: $\{(-23, 247), (-16, 161)(72, 152)(75, 112), (77, 83)\}$
   Weight: 224

   (c) Grouping: $\{(2, 88), (19, 81), (28, 76), (41, 69), (59, 61)\}$
   Weight: 120

   Justification:

   Suppose we have pairs (a, b) and (c, d) such that $a \leq c$ and $b \leq d$. The weight of these pairs is $\max\{(a+b), (c+d)\} = c+d$. If we swap b and d and make two new pairs (a, d) and (c, b), we get a pairing whose weight is $\max\{(a+d), (c+b)\}$. Since $a \leq c$, $a+d \leq c+d$ and since $b \leq d$, $c+b \leq c+d$. Thus, the new pairing has a smaller weight than the original pairing. It follows that the minimum weight pairing of our list of m numbers will never contain pairs of the form (a, b) and (c, d) with $a \leq c$ and $b \leq d$.

   An efficient way to generate a minimum weight pairing is as follows: Pair up the smallest and largest numbers in the list. Eliminate this pair and once again pair up the smallest and largest numbers from the remainder of the list, etc.

   (a) 103, 24, 77, 65, 12, 108, 69, 25, 66, 83

Best grouping:	(12,108)	(24, 103)	(25,83)	(65,77)	(66, 69)
Weights	120	127	108	142	135

   Weight: $142 = \max\{120, 127, 108, 142, 135\}$.

   (b) 83, 112, -16, 72, 161, 75, 152, -23, 77, 247

Best grouping:	(-23,247)	(-16,161)	(72,152)	(75,112)	(77,83)
Weights	224	145	224	187	160

   Weight: $224 = \max\{224, 145, 224, 187, 160\}$.

   (c) 19, 81, 2, 41, 61, 59, 28, 69, 76, 88

Best grouping:	(2,88)	(19,81)	(28,76)	(41,69)	(59,61)
Weights	90	100	104	110	120

   Weight: $120 = \max\{90, 100, 104, 110, 120\}$.

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2. A number is good precisely when it is a multiple of 4.

   Justification:

   Clearly, no odd number can be good, because the rules generate only even numbers.

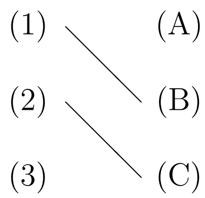
   Suppose we start with an even number. After we double it for the first time, we have a multiple of 4. After this, we either double it again (which still leaves us with a multiple of 4) or we subtract 100 (since 100 is a multiple of 4, subtracting 100 leaves us again with a multiple of 4). In other words, not only do we generate only even numbers, we generate only multiples of 4. So, if we start with an even number that is not a multiple of 4, we will never get back to the starting number.

Thus, for a number to be good it must be a multiple of 4.

We also have to prove that every multiple of 4 is indeed good. However, there are only 22 two digit multiples of 4—namely 12,16,...,96—so we can exhaustively verify that each of these is in fact good. These numbers break up into two generating cycles, as follows:

$$
\begin{array}{ccccccccccccccccccc}
12 & \rightarrow & 24 & \rightarrow & 48 & \rightarrow & 96 & \rightarrow & 92 & \rightarrow & 84 & \rightarrow & 68 & \rightarrow & 36 & \rightarrow & 72 & \rightarrow & 44 \\
\uparrow & & & & & & & & & & & & & & & & & & \downarrow \\
56 & \leftarrow & 28 & \leftarrow & 64 & \leftarrow & 32 & \leftarrow & 16 & \leftarrow & 8 & \leftarrow & 4 & \leftarrow & 52 & \leftarrow & 76 & \leftarrow & 88
\end{array}
\qquad
\begin{array}{ccc}
20 & \rightarrow & 40 \\
\uparrow & & \downarrow \\
60 & \leftarrow & 80
\end{array}
$$

3.



1 is compatible with B
2 is compatible with C
3 and A are not compatible with any sequence.

Justification:

Input sequences	*Output sequences*
(1) 5,8,10,3,2,9,7,6,4,1	(A) 6,2,7,4,5,8,3,10,9,4
(2) 10,1,9,6,5,4,2,8,3,7	(B) 3,9,7,2,10,6,1,4,8,5
(3) 7,6,2,5,3,4,8,10,4,9	(C) 9,5,6,1,2,4,10,7,3,8

Since only 3 and A have two 4's and no 1, we can break up the problem into checking the compatibility of 1 and 2 against B and C and 3 against A.

Consider an input sequence 1,2,3. The only output sequence that is *not* compatible with 1,2,3 is 3,1,2. More generally, if the input sequence contains 3 numbers $\ldots a \ldots b \ldots c \ldots$ where $\ldots$ stands for an arbitrary sequence of other numbers in between, and the output sequence contains $\ldots c \ldots a \ldots b \ldots$, then the output is not compatible with the input. Otherwise it is compatible.

To check whether 1 is compatible with B and C, use numbers from 11 to 20 to systematically renumber the sequence in 1 in ascending order and use the same renumbering for B and C. We renumber 1 as

1: 11(5), 12(8), 13(10), 14(3), 15(2), 16(9), 17(7), 18(6), 19(4), 20(1)

Then we have

B: 14(3), 16(9), 17(7), 15(2), 13(10), 18(6), 20(1), 19(4), 12(8), 11(5)

C: 16(9), 11(5), 18(6), 20(1), 15(2), 19(4), 13(10), 17(7), 14(3), 12(8)

Now we see if there are three numbers $11 \leq a < b < c \leq 20$ such that the numbers appear in the order c, a, b in the renumbered version of B or C.

In B, we can check that for each number c, all numbers smaller than c that appear after c appear in descending order — in other words, if we have $a < b < c$ and a, b appear after c in B, then a appears after b, so the three numbers appear as c, b, a, not c, a, b. Thus, there is no order violation in B and B is compatible with 1.

This is not the case with C. For instance, we have the sequence 16,11,15, which corresponds to illegally reordering 5,2,9 from sequence 1 as 9,5,2 in sequence C. So, C is not compatible with 1.

Similarly, we can check sequence 2 against B and C.

2: 11(10), 12(1), 13(9), 14(6), 15(5), 16(4), 17(2), 18(8), 19(3), 20(7)

B: 19(3), 13(9), 20(7), 17(2), 11(10), 14(6), 12(1), 16(4), 18(8), 15(5)

C: 13(9), 15(5), 14(6), 12(1), 17(2), 16(4), 11(10), 20(7), 19(3), 18(8)

In B, we have, for instance, the sequence 19,13,17 which corresponds to illegally reordering 9,2,3 from sequence 2 as 3,9,2 in sequence B. So, B is not compatible with 2.

In C, we can check that for each number c, all numbers smaller than c that appear after c appear in descending order so there is no order violation and C is compatible with 2.

Finally, we use the same technique to renumber and compare 3 and A.

3: 11(7), 12(6), 13(2), 14(5), 15(3), 16(4), 17(8), 18(10), 19(4), 20(9)

A: 12(6), 13(2), 11(7), 16(4), 14(5), 17(8), 15(3), 18(10), 20(9), 19(4)

A contains the illegal sequence 16,14,15 corresponding to reordering 5,3,4 from 3 as 4,5,3 so A is not compatible with 3. (Actually, we should also check the renumbering of A where the first 4 in A is renumbered 19 and the second 4 is renumbered 16. In this case, we have the illegal sequence 19,14,15, so the sequences remain incompatible).

---

4. (a) Sack contents: 10 kg of B, 10 kg of C
   Total Value: 2833.33

   (b) Sack contents: 10 kg of B, 9 kg of D, 11 kg divided in any way between A and C.
   Total Value: 3920

   (c) Sack contents: 10 kg of B, 15 kg of C, 5 kg of D
   Total Value: 3075

Justification:

The thief should choose items according to their value per unit weight. Thus, we compute the value per unit weight of each item and fill the sack starting with the highest value item.

(a) $W = 20$

	A	B	C
amount	15	10	18
value	1800	1500	2400
unit value	120	150	133.33

Therefore, pick items in the order B, C, A. After exhausting all 10 kg of B, add 10 kg of C to fill the sack. Total value is $10*150+10*133.33 = 1500+1333.33 = 2833.33$.

(b) $W = 30$

	A	B	C	D
amount	25	10	15	9
value	3000	1400	1800	1200
unit value	120	140	120	133.33

Therefore, pick items in the order B, D, and then A or C. After exhausting all 10 kg of B and all 9 kg of D, add 11 kg of A/C to fill the sack. Total value is $10 * 140 + 9 * 133.33 + 11 * 120 = 1400 + 1200 + 1320 = 3920$.

(c) $W = 30$

	A	B	C	D
amount	25	10	15	20
value	2250	1100	1500	1900
unit value	90	110	100	95

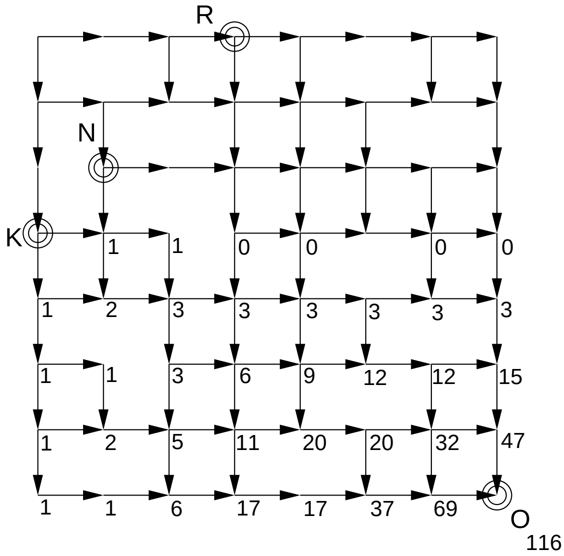
Therefore, pick items in the order B, C, D, A. After exhausting all 10 kg of B and all 15 kg of C, add 5 kg of D to fill the sack. Total value is $10*110+15*100+5*95 = 1100 + 1500 + 475 = 3075$.

---

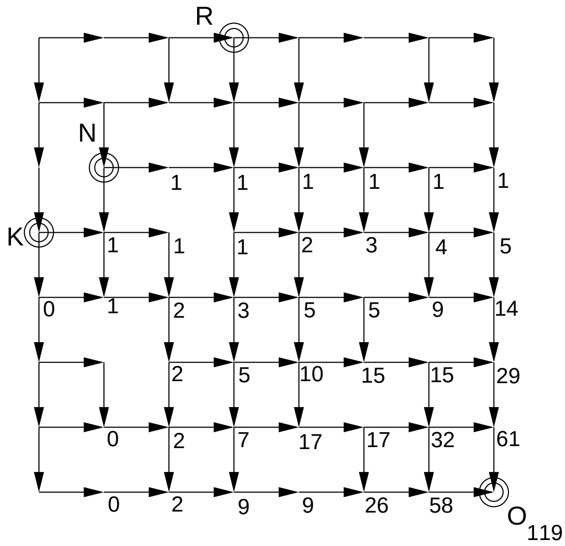
5. (a) Komal: 116
   (b) Narain: 119
   (c) Robert: 97

Justification:

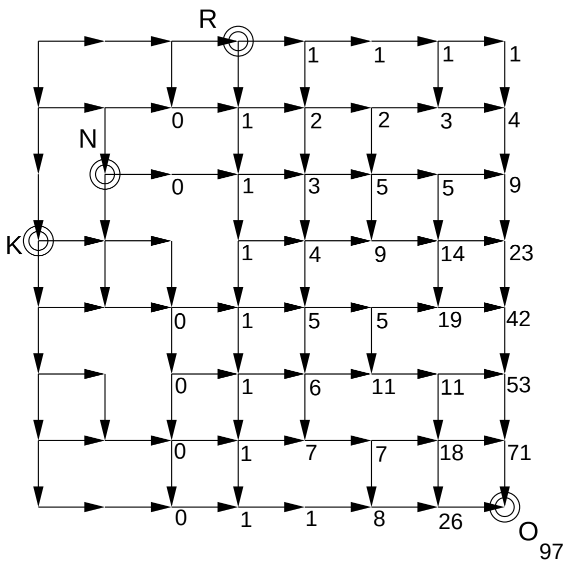
To count the number of ways to reach an intersection, we can add up the number of ways of reaching each intersection that is one step back from the current intersection. Since all the roads in the map are one way pointing down or to the right, each intersection has at most two immediately preceding intersections, above and to the left. We can thus start from the intersections marked K, N and R and systematically fill up row by row the number of ways of reaching each intersection, until we arrive at the figure we want, for the intersection marked O. The computation for each of the three cases is shown below.



Komal



Narain



Robert

Zonal Informatics Olympiad, 2004

Instructions to candidates

1. The duration of the examination is 3 hours.
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Zonal Informatics Olympiad, 2004

Questions

1. Two words are *neighbours* if you can go from one to the other by removing one letter, adding one letter or modifying one letter. For example “blow” is a neighbour of “bow” (removing one letter), “below” (adding one letter) and “brow” (modifying one letter).

   Given a set of words, the aim is to find the longest sequence of words without repetitions where each pair of consecutive words in the sequence are neighbours. There may be many sequences of maximum length. It is sufficient to identify any one.

   For instance, given the words {below, blow, bow, bowl, brow, crow, rot, row}, the longest such sequence is of length six. Actually, there are two sequences of length six in this set: “crow, row, brow, blow, bow, bowl” and “rot, row, brow, blow, bow, bowl”.

   For each of the following sets of words, write out any one such sequence of maximum length.

   (a) {be, bed, bet, bud, but, dig, do, dog, dug, get, go, god, got}

   (b) {fat, fate, fight, fit, fright, gait, gate, light, mite, quit, quite, right, sat, sigh, sight, sit, site, suite, writ, write}

   (c) {age, bat, batch, bath, bathe, bather, batter, cache, cat, catch, catcher, fat, fate, father, fresh, garb, garbage, heathen, later, lather, mash, mate, matt, matter, rash, rat, thrash, thresh, trash}

2. An ancient Indian manuscript gives the following recipe to convert any whole number $N > 1$ into a sequence of a's and b's.

   Starting with N, repeat the following steps. Stop when you reach the number 1.

   (a) If the number is even, divide it by 2 and write the letter a.

   (b) If the number is odd, add 1 to it, divide the result by 2 and write the letter b.

   For example, if we start with $N = 26$, we obtain the sequence *abbaa*, as follows:

Step	*Current number*	*Next number*	*Write out*	*Explanation*
1.	26	13	a	26 is even, $26 \div 2 = 13$
2.	13	7	b	13 is odd, $(13 + 1) \div 2 = 7$
3.	7	4	b	7 is odd, $(7 + 1) \div 2 = 4$
4.	4	2	a	4 is even, $4 \div 2 = 2$
5.	2	1	a	2 is even, $2 \div 2 = 1$
6.	1	–	–	Stop when we reach 1

   Reading the fourth column from top to bottom, we get the sequence generated by the number 26, namely *abbaa*.

This recipe converts each whole number larger than 1 to a unique sequence of a's and b's. The problem is to invert the recipe and determine for each of the following sequences of a's and b's, the corresponding whole number that yields that sequence when the recipe is applied to it. For instance, for *abbaa*, the answer is 26.

(a) *bbaaba*

(b) *babbabba*

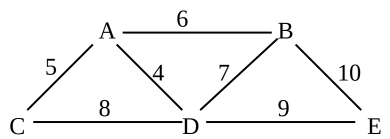
(c) *ababaaabaa*

3. A collection of cities are connected by a network of ordinary two-way roads. Not all pairs of cities are directly connected, but it is possible to go from any city to any other city by taking a sequence of roads.

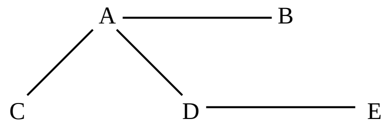
The government has decided to upgrade some of these roads to expressways. The goal is to upgrade enough roads so that any pair of cities is connected by a sequence of expressways. However, the government also wants to keep the cost of this operation low, so it wants to minimize the total length of road that is upgraded to expressway.

Given a collection of cities and the roads between them, along with the length of each road, determine which roads should be upgraded to meet the government objective.

For instance, consider the network of roads on the right connecting five cities A, B, C, D and E. Each line represents a road and the number next to the line represents the length of the road.

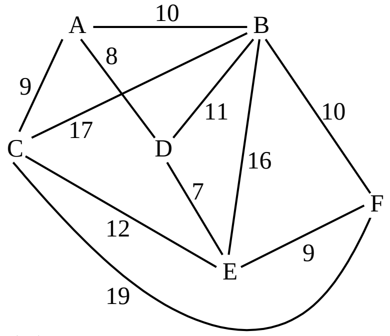


The shortest collection of roads to upgrade to expressways in this network is shown on the right:

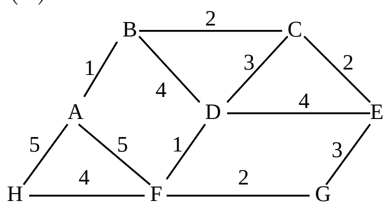


Find the shortest collection of roads to convert to expressways in the following networks. There may be more than one solution possible. It is sufficient to provide any one valid solution.

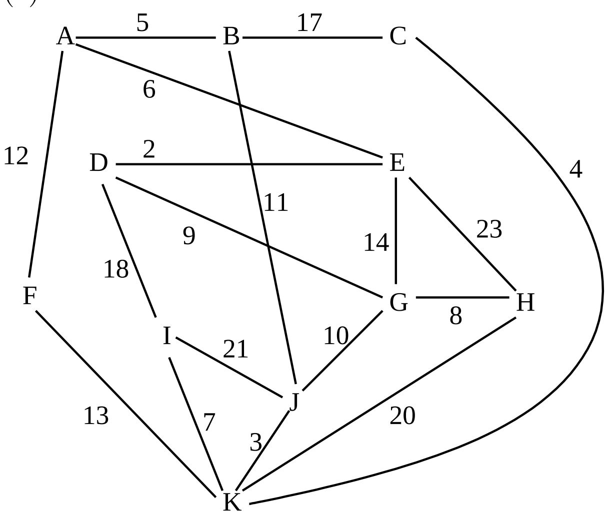
(a)



(b)



(c)



4. In a warehouse, there are a number of empty rectangular cartons of different sizes. The cartons are open at the top, so smaller cartons can be stacked inside bigger cartons.

   One carton can be stacked inside another one provided each of its sides is less than or equal to the corresponding side of the other—for instance, a 10×8 carton can be stacked inside a 9×12 carton or a 10×10 carton, but not inside a 9×9 carton. Notice that you are allowed to rotate the cartons before stacking them.

   The heights of the cartons do not matter when stacking one inside the other. There is no limit to how many cartons can be stacked one inside the other.

   You are given a collection of cartons and their sizes. You have to decide how to stack them one inside the other so that they occupy the minimum floor area.

   Example: You have five cartons labelled A, B, C, D and E as follows:

   $A : 10 \times 13 \quad B : 11 \times 9 \quad C : 12 \times 8 \quad D : 7 \times 6 \quad E : 13 \times 7$

   One possible stacking is D inside B inside A, with C and E separate. This occupies floor area $A + C + E = 130 + 96 + 91 = 317$. Another possible stacking is D inside C inside A, with B and E separate. This occupies floor area $A+B+E = 130+99+91 = 320$. Thus, the first arrangement is better.

   For each of the following lists of boxes, determine the stacking arrangement that occupies the minimum floor area. In your answer, list out the way the cartons should be stacked. For example, here is how the solution looks for the example above:

   D in B in A ; C ; E

   (a) $A : 10 \times 11 \quad B : 9 \times 12 \quad C : 8 \times 13 \quad D : 11 \times 9 \quad E : 7 \times 12$
   $F : 11 \times 11 \quad G : 12 \times 10 \quad H : 9 \times 14 \quad I : 15 \times 8$

   (b) $A : 18 \times 8 \quad B : 9 \times 15 \quad C : 10 \times 11 \quad D : 21 \times 9 \quad E : 10 \times 17$
   $F : 7 \times 16 \quad G : 8 \times 13 \quad H : 11 \times 9 \quad I : 10 \times 10$

   (c) $A : 25 \times 18 \quad B : 17 \times 30 \quad C : 33 \times 15 \quad D : 16 \times 25 \quad E : 15 \times 30$
   $F : 31 \times 14 \quad G : 19 \times 27 \quad H : 18 \times 30 \quad I : 33 \times 17 \quad J : 19 \times 30$
   $K : 18 \times 33 \quad L : 24 \times 15 \quad M : 8 \times 30$

5. We go back a long time ago to a galaxy far, far away. The planets Aleph and Gimmel are connected by teleports that permit instantaneous, interplanetary transportation of people between the two planets.

   Each teleport on one planet has a single one-way connection to a teleport on the other planet. Every teleport has exactly one such outgoing connection but may have any number of incoming connections—in fact, it may have no incoming connections at all. For instance, teleports A, B and C on Aleph may have outgoing connections to teleports X, Y and Z on Gimmel, respectively, but in the reverse direction teleport X's outgoing connection may be to to teleport B, while teleports Y and Z on Gimmel both have outgoing connections to teleport A. Thus, on Aleph, A has two incoming connections while C has none.

Each teleport can work in one of two modes, *Receiving* and *Sending*. A teleport in *Receiving* mode can only be used for inward transportation while a teleport in *Sending* mode can only be used for outward transportation. For instance, suppose that A has an outgoing connection to X and incoming connections from Y and Z. If the mode of A is set to *Receiving*, it can receive people from both Y and Z, but it cannot send out people to X. On the other hand, if mode of A is set to *Sending*, it can send people to X, but it cannot receive people from Y or Z. A connection from teleport A on one planet to teleport B on the other planet can be used only if the mode of A is *Sending* and the mode of B is *Receiving*.

The goal is to set the mode of each teleport to either *Receiving* or *Sending* so that all teleports remain usable. In other words, we want the following conditions to be satisfied:

- If teleport X is set to *Sending* mode, then the teleport to which X has an outgoing connection is set to Receiving mode.
- If teleport X is set to *Receiving* mode, then there is at least one teleport Y with an outgoing connection to X such that Y is set to *Sending* mode.

There may be more than one consistent mode setting. It is sufficient to find any one.

Example: Aleph has four teleports $A_1, \ldots, A_4$. Gimmel has five teleports $G_1, \ldots, G_5$. The connections are:

$A_1 \to G_3$, $A_2 \to G_5$, $A_3 \to G_2$, $A_4 \to G_5$.
$G_1 \to A_4$, $G_2 \to A_4$, $G_3 \to A_4$, $G_4 \to A_1$, $G_5 \to A_3$.

Here is a consistent setting of the modes (R indicates *Receiving* mode, S indicates *Sending* mode).

$A_1 = R$, $A_2 = S$, $A_3 = S$, $A_4 = R$
$G_1 = S$, $G_2 = R$, $G_3 = S$, $G_4 = S$, $G_5 = R$

Determine a consistent mode setting for each of the following collections of teleports.

(a) Aleph has seven teleports $A_1, \ldots, A_7$. Gimmel has four teleports $G_1, \ldots, G_4$.
$A_1 \to G_1$, $A_2 \to G_2$, $A_3 \to G_3$, $A_4 \to G_3$, $A_5 \to G_1$, $A_6 \to G_2$, $A_7 \to G_4$.
$G_1 \to A_5$, $G_2 \to A_7$, $G_3 \to A_4$, $G_4 \to A_6$.

(b) Aleph has five teleports $A_1, \ldots, A_5$. Gimmel has eight teleports $G_1, \ldots, G_8$.
$A_1 \to G_5$, $A_2 \to G_2$, $A_3 \to G_4$, $A_4 \to G_1$, $A_5 \to G_8$.
$G_1 \to A_3$, $G_2 \to A_2$, $G_3 \to A_5$, $G_4 \to A_4$, $G_5 \to A_1$, $G_6 \to A_5$, $G_7 \to A_2$, $G_8 \to A_2$.

(c) Aleph has six teleports $A_1, \ldots, A_6$. Gimmel has nine teleports $G_1, \ldots, G_9$.
$A_1 \to G_9$, $A_2 \to G_4$, $A_3 \to G_5$, $A_4 \to G_5$, $A_5 \to G_8$, $A_6 \to G_7$.
$G_1 \to A_5$, $G_2 \to A_1$, $G_3 \to A_5$, $G_4 \to A_4$, $G_5 \to A_5$, $G_6 \to A_4$, $G_7 \to A_6$, $G_8 \to A_1$, $G_9 \to A_2$.

Zonal Informatics Olympiad, 2004: *Answer sheet*

Roll No:	**Examination Centre:**

Write only your final answers in the space provided. Write all rough work on separate sheets.

1. (a)

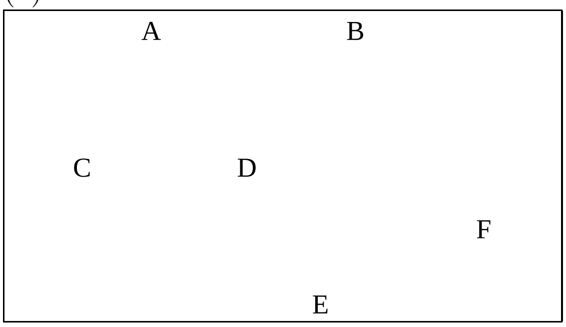
   (b)

   (c)

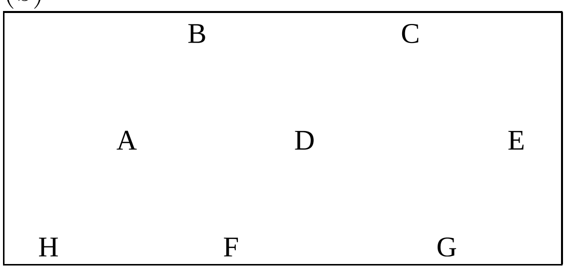
2. (a) (b) (c)

3. Draw lines corresponding to the roads that should become expressways.

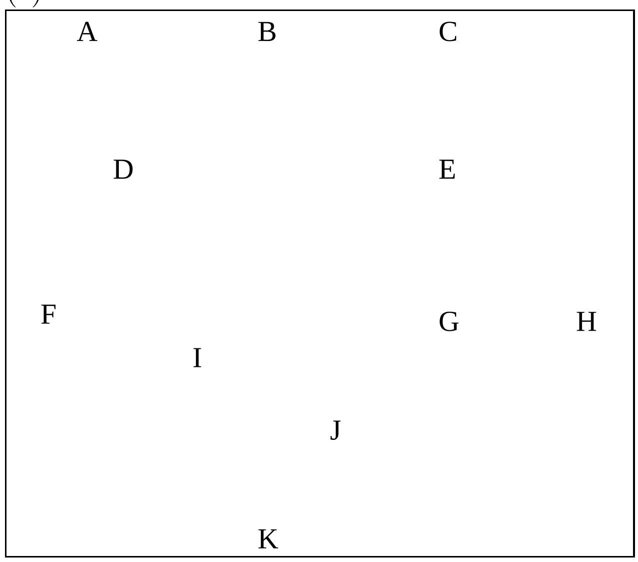
(a)



(b)



(c)



4. (a)

   (b)

   (c)

5. (a) $A_1 =$, $A_2 =$, $A_3 =$, $A_4 =$, $A_5 =$, $A_6 =$, $A_7 =$
   $G_1 =$, $G_2 =$, $G_3 =$, $G_4 =$

   (b) $A_1 =$, $A_2 =$, $A_3 =$, $A_4 =$, $A_5 =$
   $G_1 =$, $G_2 =$, $G_3 =$, $G_4 =$, $G_5 =$, $G_6 =$, $G_7 =$, $G_8 =$

   (c) $A_1 =$, $A_2 =$, $A_3 =$, $A_4 =$, $A_5 =$, $A_6 =$
   $G_1 =$, $G_2 =$, $G_3 =$, $G_4 =$, $G_5 =$, $G_6 =$, $G_7 =$, $G_8 =$, $G_9 =$

For official use only. Do not write below this line.

1.
(a)	(b)	(c)

2.
(a)	(b)	(c)

3.
(a)	(b)	(c)

4.
(a)	(b)	(c)

5.
(a)	(b)	(c)

Total

Zonal Informatics Olympiad, 2004

Solutions

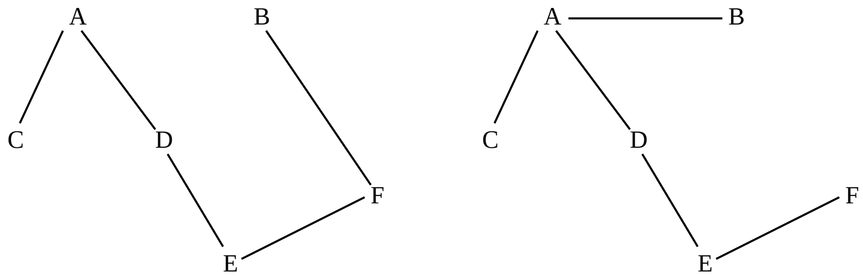
1. (a) $$\left.\begin{array}{l}\text{but–bud–bed–be–}\\ \text{be–bed–bud–but–}\end{array}\right\}\text{–bet–get–got–god–go–do–dog–}\left\{\begin{array}{l}\text{–dig–dug}\\ \text{–dug–dig}\end{array}\right.$$

   (b) $$\text{quit–quite–suite–site–sit–}\left\{\begin{array}{c}\text{–sat–}\\ \text{–fit–}\end{array}\right\}\text{–fat–fate–gate}$$

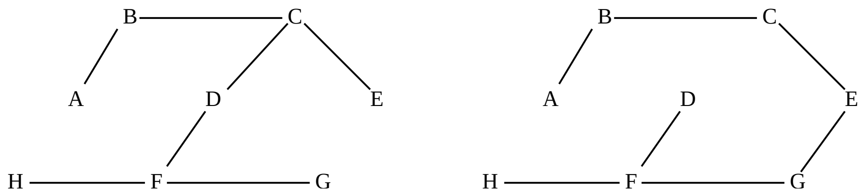
   (c) $$\text{later–lather–father–bather–bathe–bath–bat–}\left\{\begin{array}{c}\text{–cat–rat–}\\ \text{–rat–cat–}\end{array}\right\}\text{–fat–fate–mate–matt}$$

2. (a) 45 (b) 147 (c) 886

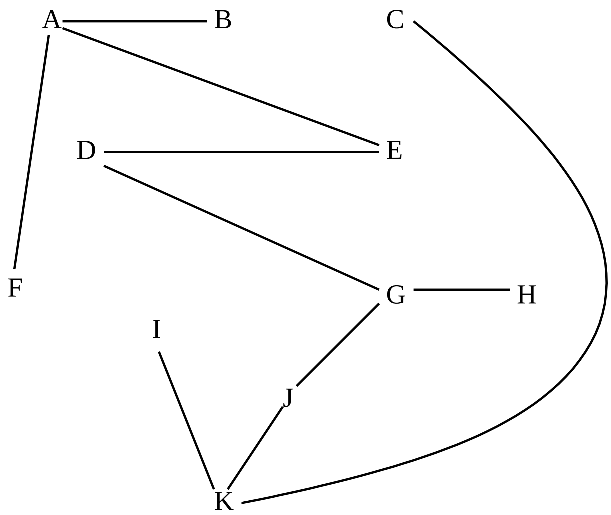
3. (a)



(b)

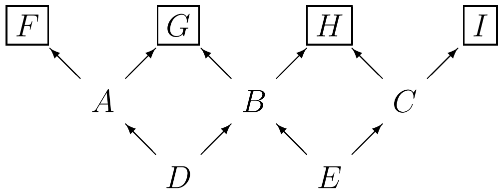


(c)



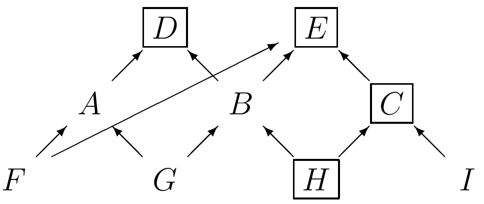
4. An arrow in the diagram shows that the carton below can fit inside the carton above. Boxed items are cartons that *must* be outermost in any valid solution.

(a)



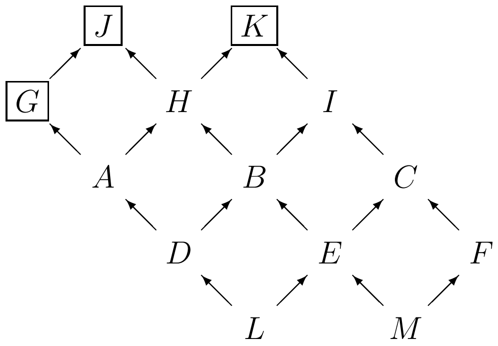
D in A in F, E in B in G, C in H, I
D in A in F, E in B in G, H, C in I
D in A in F, G, E in B in H, C in I
D in A in F, G, B in H, E in C in I
F, D in A in G, E in B in H, C in I
F, D in A in G, B in H, E in C in I
F, A in G, D in B in H, E in C in I
...
(Any valid packing where outermost are F, G, H, I)

(b)



H
I in C
F in A in D
G in B in E

(c)



L in D in A in G
M in E in B in H in J
F in C in I in K

L in D in A in G
E in B in H in J,
M in F in C in I in K

D in A in G
L in E in B in H in J
M in F in C in I in K

5. (a) $A_1 = S$, $A_2 = S$, $A_3 = S$, $A_4 = S$, $A_5 = S$, $A_6 = S$, $A_7 = S$
$G_1 = R$, $G_2 = R$, $G_3 = R$, $G_4 = R$

(b) $A_1 = \mathbf{X}$, $A_2 = R$, $A_3 = \mathbf{Y}$, $A_4 = \mathbf{Y}$, $A_5 = R$
$G_1 = \overline{\mathbf{Y}}$, $G_2 = S$, $G_3 = S$, $G_4 = \overline{\mathbf{Y}}$, $G_5 = \overline{\mathbf{X}}$, $G_6 = S$, $G_7 = S$, $G_8 = S$
where $\mathbf{X} = \neg\overline{\mathbf{X}}$, $\mathbf{Y} = \neg\overline{\mathbf{Y}}$, (i.e., $\mathbf{X} = S$ iff $\overline{\mathbf{X}} = R$, $\mathbf{Y} = S$ iff $\overline{\mathbf{Y}} = R$)

(c) $A_1 = R$, $A_2 = R$, $A_3 = S$, $A_4 = R$, $A_5 = R$, $A_6 = \mathbf{X}$
$G_1 = S$, $G_2 = S$, $G_3 = S$, $G_4 = S$, $G_5 = R$, $G_6 = S$, $G_7 = \overline{\mathbf{X}}$, $G_8 = S$, $G_9 = S$
where $\mathbf{X} = \neg\overline{\mathbf{X}}$.

Zonal Informatics Olympiad, 2005

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Zonal Informatics Olympiad, 2005

Questions

1. The director of Hind Circus has decided to add a new performance called the monkey dance to his show. The monkey dance is danced simultaneously by N monkeys.

   There are N circles drawn on the ground. There are N arrows drawn between the circles in such a way that for each circle, exactly one arrow begins at that circle and exactly one arrow ends at that circle. No arrow can both begin and end at the same circle.

   When the show begins, each monkey sits on a different circle. At each whistle of the ringmaster, all the monkeys simultaneously jump from one circle to the next, following the arrow leading out of the current circle. This is one step of the dance. The dance ends when all the monkeys have *simultaneously* returned to the circles where they initially started.

   The director wishes the dance to last as many steps as possible. This can be achieved by drawing the arrows intelligently.

   For each of the three values of N given below, what is the maximum number of steps that the monkey dance can be made to last by drawing arrows appropriately?

   (a) 9 (b) 12 (c) 15

2. A bus company operates a number of routes connecting different bus stops in the city. Buses run in both directions along each route. The bus company hires a supervisor who has to periodically check that the signboards and waiting areas at each bus stop are in good condition.

   The supervisor needs to set up a central office at one of the bus stops. He would like to locate this office at a bus stop from where the maximum number of stops he has to travel to reach any other bus stop is minimized.

   For simplicity, the bus stops are identified by numbers rather than names. Bus routes are described by the sequence of bus stops that they pass through. Suppose, for example that we had the following set of bus stops and bus routes.

Bus stops	1,2,. . . ,6
Route A	1—2—3
Route B	2—4—7
Route C	2—4—5—6

   In this case the ideal location for the supervisor's office is bus stop 4, because every other bus stop is at most two stops away. Notice that from bus stop 2, the supervisor can reach every other bus stop without changing buses, but bus stop 4 is better than bus stop 2 because from bus stop 2 it takes three stops to reach bus stop 6. The supervisor *does not* care about how many time he has to change buses; he only wants to minimize the maximum number of stops to every other bus stop.

In the three problems below, you are given a set of bus stops and bus routes. Your task is to identify the ideal location for the supervisor's central office in each case.

(a)

Bus stops	1,2,...,15
Route A	5—4—2—1—6—8—14—15
Route B	3—2—1—6—7—9—11—12
Route C	10—9—7—6—8—13

(b)

Bus stops	1,2,...,20
Route A	9—4—2—1—15—16—17—18—19—20
Route B	10—4—2—1—3
Route C	1—2—5—11
Route D	12—13—6—3—8
Route E	14—6—3—7

(c)

Bus stops	1,2,...,24
Route A	3—1—2—24
Route B	1—4—5—6—7—10—14—15—17
Route C	16—15—14—13—11
Route D	12—13—14—10—7—8
Route E	9—7—6—19—22
Route F	23—19—6—18—21
Route G	20—18—6—5—4

3. There is a unique sequence of positive numbers that starts with the number 1 at the first position and has the following properties.

   - Each value in the sequence is greater than or equal to every value that appears earlier in the sequence.
   - If the value at position k in the sequence is m, then the number k appears exactly m times in the sequence.

   The first few numbers in this sequence are

Position	:	1	2	3	4	5	6	7	8	9	10	11	12	⋯
Sequence value	:	1	2	2	3	3	4	4	4	5	5	5	6	⋯

   Notice, for instance, that the value at position 4 in the sequence is 3, so 4 itself appears 3 times in the sequence.

What is the value that appears at each of the following positions in this sequence?

(a) 411 (b) 1000 (c) 1245

4. A teacher assigns homework at the beginning of the first day of class. Each homework problem carries one mark, plus a bonus if submitted within a specified number of days. Each homework problem takes exactly one day to complete.

The deadline for earning the bonus and the number of bonus marks are different for each homework problem. For instance, if a problem has a deadline of 6 days and carries bonus 10 marks, then it earns 10 bonus marks provided it is submitted before the end of the 6th day.

For example, suppose there are seven problems with deadlines and bonuses as follows:

Problem name	a	b	c	d	e	f	g
Deadline for bonus	1	1	3	3	2	2	6
Bonus marks	6	7	2	1	4	5	1

Then the maximum number of bonus marks one can obtain is 15, which can be achieved by completing the problems in the sequence b, f, c, a, g, e, d. Note that there are also other sequences that achieve the same effect.

Your task is to find a schedule to complete all homework problems so as to maximize bonus marks for the following three sets of data.

(a)

Problem name	a	b	c	d	e	f	g	h	i	j	k	ℓ
Deadline for bonus	9	4	1	4	2	3	6	3	2	5	5	8
Bonus marks	2	6	2	5	3	3	5	7	4	6	4	6

(b)

Problem name	a	b	c	d	e	f	g	h	i	j	k	ℓ	m	n	o
Deadline for bonus	2	4	5	2	6	8	3	9	4	9	2	5	6	7	9
Bonus marks	4	3	5	3	7	7	4	8	5	6	6	4	5	6	2

(c)

Problem name	a	b	c	d	e	f	g	h	i	j	k	ℓ	m	n	o
Deadline for bonus	6	2	3	14	10	4	5	5	7	8	6	7	8	3	9
Bonus marks	5	4	3	2	2	3	4	1	6	7	3	4	7	6	6

5. A skier wants to ski down from the top of a mountain to its base. There are several possible routes, using different slopes en route, and passing through some flat areas.

For each slope, there is a maximum advisable speed. The effort required to ski down a slope depends upon the length of the slope and the speed of skiing and is given by the following formula: $e = d \times (60 - s)$ if $s \leq 50$, and $e = d \times (s - 40)$ if $s > 50$, where e is the effort required, d is the distance travelled, and s is the speed of travel. Travelling across a flat area requires no effort.

In each of the problems below, you are given a map of the mountain slopes—that is, the list of flat areas and details about the slopes connecting these areas. Note that one can only ski downwards on a slope. For each slope, you are given the flat areas that it connects, the length of the slope and the maximum advisable speed for it.

You have to determine the minimum total effort that the skier has to expend in order to reach the mountain base, while staying within the maximum advisable speed at every slope.

(a) There are 7 flat areas and 9 slopes connecting them. The flat areas are numbered from 1 to 7 with flat area 1 being at the top and flat area 7 at the bottom. The data for the 9 slopes is:

Slope No	1	2	3	4	5	6	7	8	9
From flat	1	1	1	2	3	3	4	5	6
To flat	2	3	4	5	5	6	6	7	7
Max speed	60	40	20	40	60	30	20	30	30
Length	15	10	05	20	30	10	05	10	10

(b) There are 9 flat areas and 12 slopes connecting them. The flat areas are numbered from 1 to 9 with flat area 1 being at the top and flat area 9 at the bottom. The data for the 12 slopes is:

Slope No	1	2	3	4	5	6	7	8	9	10	11	12
From flat	1	1	2	2	3	3	4	5	5	6	7	8
To flat	2	3	4	5	5	6	7	7	8	8	9	9
Max speed	50	40	40	50	50	40	30	60	40	40	40	20
Length	15	25	30	25	20	10	10	10	05	20	10	20

(c) There are 11 flat areas and 16 slopes connecting them. The flat areas are numbered from 1 to 11 with flat area 1 being at the top and flat area 11 at the bottom. The data for the 16 slopes is:

Slope No	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
From flat	1	1	2	2	3	3	4	4	5	5	6	6	7	8	9	10
To flat	2	3	4	5	5	6	7	8	8	9	9	10	11	11	11	11
Max speed	60	30	50	60	50	40	30	50	60	40	40	50	40	20	40	30
Length	10	20	20	10	10	10	10	30	30	20	20	40	20	10	20	10

Zonal Informatics Olympiad, 2005: *Answer sheet*

Name:	**Class:**
School:	
Examination Centre:	

Write only your final answers in the space provided. Write all rough work on separate sheets.

1. (a) Length of longest dance: ☐ (b) Length of longest dance: ☐

   (c) Length of longest dance: ☐

2. (a) Location of supervisor's office: ☐ (b) Location of supervisor's office: ☐

   (c) Location of supervisor's office: ☐

3. (a) Value at position 411: ☐ (b) Value at position 1000: ☐

   (c) Value at position 1245: ☐

4.

	Bonus marks earned	*Schedule*
(a)		
(b)		
(c)		

5. (a) Minimum effort required: ☐ (b) Minimum effort required: ☐

   (c) Minimum effort required: ☐

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1.	(a)	(b)	(c)	2.	(a)	(b)	(c)	3.	(a)	(b)	(c)
4.	(a)	(b)	(c)	5.	(a)	(b)	(c)		**Total**		

Zonal Informatics Olympiad, 2005: *Answer sheet*

Name:	**Class:**
School:	
Examination Centre:	

Write only your final answers in the space provided. Write all rough work on separate sheets.

1. (a) Length of longest dance: **20**
   (b) Length of longest dance: **60**
   (c) Length of longest dance: **105**

2. (a) Location of supervisor's office: **6**
   (b) Location of supervisor's office: **15**
   (c) Location of supervisor's office: **6**

3. (a) Value at position 411: **50**
   (b) Value at position 1000: **86**
   (c) Value at position 1245: **98**

4.

	Bonus marks earned	*Schedule*[a]														
(a)	**41**	i	h	b	d	j	g	l	a	c	e	f	k			
(b)	**55**	k	i	c	e	m	n	f	h	j	a	d	g	b	l	o
(c)	**53**	b	n	g	a	i	l	m	j	o	e	d	c	f	h	k

5. (a) Minimum effort required: **700**
   (b) Minimum effort required: **700**
   (c) Minimum effort required: **900**

[a]Note: There are other feasible schedules yielding the same number of bonus marks.